Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357501_OC**
Name of Paper	: **DSE-Numerical Methods**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Let a be a nonzero real number. For any x_0, the recursive sequence defined by $x_{n+1} = (a^3 + +2x_n{}^3)/3x_n{}^2$ converges to a. Determine the order of convergence and the asymptotic error constant.

   Find the root of the equation $\tan x + x = 0$ correct to two decimal places which lies between 2 and 2.1 using bisection method.

   Find the smallest root of the equation $x^3 - 6x^2 + 11x - 6 = 0$ by Regula-Falsi method. Perform three iterations.

2. Using scaled partial pivoting during the factor step, find matrices L, U and P such that $LU = PA$, where

$$A = \begin{bmatrix} 2 & -6 & 10 \\ 1 & 5 & 1 \\ -1 & 15 & -5 \end{bmatrix}$$

   Perform two iterations for finding the root of $f(x) = x^3 - 13$ by secant method starting with $p_0 = 3$ and $p_1 = 2$. Further, compute the ratio $\frac{|p_3 - p|}{|p_2 - p|^{1.618}}$ and show that this value approaches $\frac{|f''(p)|}{|2f'(p)|}$ with $p = \sqrt[3]{13}$.

   Find the positive root of the equation $x^2 + e^{-x} - 5 = 0$ using the Newton's method with $p_0 = 2$. Perform three iterations.

3. Solve the following system of equations using Gauss Jacobi iteration method:

$$4x_1 + 2x_2 + x_3 = -2$$
$$2x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 6x_3 = 10$$

   Take $X^{(0)} = [0.5, -0.5, -0.5]^T$ and iterate three times.

   Solve the following system of equations using SOR iteration method:

$$2x_1 - x_2 + x_3 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$x_1 - x_2 + 2x_3 = 1$$

   Take $\omega = 0.7$ with $X^{(0)} = [0,0,0]^T$ and iterate three times.

Write the following data in the divided difference tabular form and determine the missing values:

$$x_0 = 0, \quad x_1 = 1, \quad x_2 = 2, \quad x_3 = 3$$

$$f[x_0] = 2, \quad f[x_1] = 6, \quad f[x_2] = 6$$

$$f[x_0, x_1] = 4, \quad f[x_2, x_3] = 0\,, \quad f[x_1, x_2, x_3] = 0$$

Also, write the Newton form of the interpolating polynomial for the data and estimate the value at $x = 1.5$.

4. Solve the following system of equations using Gauss Seidel iteration method:

$$2x_1 - x_2 = 1$$

$$-2x_1 + 4x_2 - x_3 = 0$$

$$-x_2 + 2x_3 + x_4 = 0$$

$$-x_3 + x_4 = 1$$

Take $X^{(0)} = [0,0,0]^T$ and iterate three times.

Obtain the interpolating polynomial using Lagrange interpolating formula for the following data:

(-2, -8), (-1, -6), (0, 5), (1, 10), (2, 20), (3, 50)

Hence, estimate the value of the polynomial at $x = 0.5$ and $x = 2.5$.

5. State the Simpson's Rule for finding the integral of a continuous function $f(x)$ over a closed interval [a, b] and use it to find the approximate value of the definite integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. Also obtain the exact integral and find the absolute error. Further, state the theoretical error bound for the Simpson's Rule and verify that the theoretical error bound holds in this case.

Approximate the derivative of $f(x) = \text{Cos}\, x$ at $x_0 = 0$ using the formula

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

taking the values of h as 1, 0.1 and 0.01. Also find the error corresponding to each value of h.

6. Approximate the second derivative of $f(x) = e^{2x}$ at $x_0 = 0$ using the formula

$$f''(x_0) = \frac{f(x_0) - 2\, f(x_0 + h) + f(x_0 + 2h)}{h^2}$$

taking the values of h as 0.1, 0.01 and 0.001. Also find the error corresponding to each value of h.

Obtain an approximate solution of the Initial Value Problem (IVP)

$$\frac{dx}{dt} = \frac{\sqrt{1-x^2}}{t}, \quad 1 \le t \le 5$$

$$x(1) = 0$$

in 4 steps using Euler's method. Also find the absolute error at each step given that the exact solution of the IVP is $x(t) = Sin(\ln t)$.